Virtual Education Lab: How to design a 2D QRD

We have previously described how to design a 1D or single-plane QRD and PRD. While this is the preferred diffuser design for some applications, where the scattering is directed into a hemi-disc, like recording control rooms and home theaters, there is still a need for a diffuser that scatters incident sound into a hemisphere. This can be achieved by forming 2D or two-plane diffusers. A comparison between a 1D and 2D diffuser polar response is shown in Figure 1. The 1D diffusor scatters incident sound specularly into a hemi-disc, whereas the 2D diffusor scatters incident sound into a hemisphere, for all angles of incidence. The 2D QRD sequence generating function is an amplitude modulation of two 1D sequences, as shown in Equation 1.

$$S_{n,m} = (n^2 + m^2) \bmod N \qquad Eq.1$$

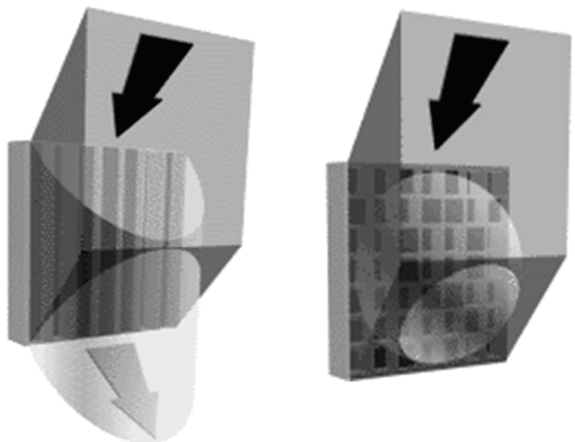

Figure 1. Left: Hemi-disc scattering from a 1D diffuser; Right: Hemispherical scattering from a 2D diffuser.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	1	4	2	2	4	1	0	1	4	2	2	4	1
1	1	2	5	3	3	5	2	1	2	5	3	3	5	2
2	4	5	1	6	6	1	5	4	5	1	6	6	1	5
3	2	3	6	4	4	6	3	2	3	6	4	4	6	3
4	2	3	6	4	4	6	3	2	3	6	4	4	6	3
5	4	5	1	6	6	1	5	4	5	1	6	6	1	5
6	1	2	5	3	3	5	2	1	2	5	3	3	5	2
7	0	1	4	2	2	4	1	0	1	4	2	2	4	1
8	1	2	5	3	3	5	2	1	2	5	3	3	5	2
9	4	5	1	6	6	1	5	4	5	1	6	6	1	5
10	2	3	6	4	4	6	3	2	3	6	4	4	6	3
11	2	3	6	4	4	6	3	2	3	6	4	4	6	3
12	4	5	1	6	6	1	5	4	5	1	6	6	1	5
13	1	2	5	3	3	5	2	1	2	5	3	3	5	2

Figure 2. Sequence elements for four periods of a 7x7 2D QRD, based on N=7.

The attenuation of the diffuse energy from a 1D N=7 diffuser is $10\log(1/7) = -8.5\,dB$ whereas the attenuation from a 7x7 2D diffuser is $10\log(1/49) = -17\,dB$ since the energy is being distributed hemispherically.

The sequence elements, S_n, calculated with Eq. 1 for four periods of a 7x7 2D QRD diffuser are shown in Figure 2, arbitrarily starting with n and m equal to zero. Since the 2D QRD is periodic, n and m can start with any value between N=0 and 7. The integer indices n and m are shown in the first column and the first row. It can be seen that the diffuser is periodic in two directions. Let's review that the design frequency, f_0, is determined with the formula:

$$f_0 = \frac{S_{\max}}{N}\frac{c}{2d_{\max}}$$

Therefore, 2D number theoretic diffusers will have less bass efficiency than a 1D diffuser, because the ratio S_{max}/N tends to be close to 1 for 2D diffusers. For example, S_{max}/N for a 1D QRD is 4/7, whereas S_{max}/N for a 2D QRD is 6/7.

Figure 2 illustrates that other sequences can be used. There is a blue outlined diagonal sequence (4, 1, 2, 0, 2, 1, 4) shown in Figure 2. This is the original sequence (0, 1, 4, 2, 2, 4, 1), but with every fourth element used. This new sequence has the same Fourier properties as the original sequence does due to the shift properties of quadratic residue sequences. This indicates that this 2D diffuser will have good scattering in the directions along the diagonals, as well as in orthogonal directions. As we proceed in this educational curriculum, we will show actual animated gif 3D polar pattern simulations at all frequencies, using the new wave-based BEM virtual goniometer program called VIRGO, which we have described in previous weekly posts (DR 220602-DR 220630).

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