Virtual Education Lab: How to design a 1D primitive root diffuser

In the last two posts, we discussed how to design a QRD and described some of its properties and limitations. In this post, we will explain how to design another popular 1D diffuser based on a primitive root (PR) sequence, the primitive root diffuser (PRD). We begin with the PR generating sequence.

$$S_n = r^n \bmod N;\ \ n = 1, 2, 3..N-1 \qquad \text{Eq. 1}$$

Where S_n are the PR sequence numbers, N is an odd prime and r is a primitive root of N. A primitive root is one for which all S_n values are unique. The PRD will have N-1 wells per period. Equation 1 can be rewritten as a recursive relationship, because Eq. 1 can cause overflow problems when being computed for large values of N.

$$S_n = (r.S_{n-1}) \bmod N \qquad \text{Eq. 2}$$

Table 1. Primitive roots, r, for a few prime numbers, N

N	7	11	13	17	19
r	3	2	2	3	2

In Table 1, we list the primitive roots, r, for a few prime numbers, N. In Table 2, we calculate the PR sequence for primes 7 and 11. It can be seen that the PR sequence contains all of the numbers from 1 to N-1 in a special jumbled order. The PR sequence for N=7 (3, 2, 6, 4, 5, 1) is also periodic (blue sequence numbers) as with the QR. One full repeat of the N=7 PR sequence is shown, but only the first two repeated sequence numbers for N=11 are shown for brevity. However, the PR sequence does not contain the mirror symmetry of the N=7 QR sequence (0, 1, 4, 2, 2, 4, 1). The well depths, d_n are defined similarly to the QRD in Eq. 3, except N is replaced by (N-1).

$$d_n = \frac{S_n c}{2(N-1)f_0} \qquad \text{Eq. 3}$$

Table 2. PR sequences for prime 7, primitive root 3 and prime 11, primitive root 2.

n	1	2	3	4	5	6	7	8	9	10	11	12
3^n mod 7	3	2	6	4	5	1	**3**	**2**	**6**	**4**	**5**	**1**
n	1	2	3	4	5	6	7	8	9	10	11	12
2^n mod 11	2	4	8	5	10	9	7	3	6	1	**2**	**4**

Figure 1. Example of two periods of an N=7 PRD.

The PRD is meant to reduce the energy of the specular reflection direction and so produce a notch diffuser. While this is true at discrete frequencies, the PRDs are impractical notch diffusers, but offer good performance and an interesting aesthetic. This is particularly true for the 2D diffuser, which we will discuss in subsequent posts.

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