Virtual Education Lab: How to design a 2D PRD

We have previously described how to design a 2D or two-plane QRD. In this post, we design a 2D primitive room diffuser, PRD. We will use a folding technique called the Chinese remainder theorem. This approach folds a 1D PR sequence into a 2D PR sequence and preserves the good autocorrelation and Fourier properties. To use this method, the 1D PR sequence prime, N, must have two factors of N-1 that are coprime. By coprime, it is meant that the only common factor for the two umbers is 1. For example, this works for N=7 (3x2), 11 (5x2), 13 (4x3), etc. But not for N=17, since N-1=16 does not have two coprimes. For practical architectural purposes coprimes that are nearly the same make a good choice, because by slightly adjusting the block or well width you can create a square panel. Let's explore N=7, where the primitive root, r, is 3. Using Eq. 1, we generate the PR sequence numbers of (3, 2, 6, 4, 5, 1). The two coprimes of N-1=6 are 2x3. Let's also recall that these sequences are periodic. 1D sequences are periodic in one plane and 2D sequences are periodic in two planes. In Table 1, we create an array of 6 periodic 2x3 matrices and place the 1D PR N=7 sequence along the diagonal (S1, S2, S3, S4, S5, S6). Let's begin to fill the upper left base 2x3 array, outlined with a thick border, with sequence numbers along the diagonal. S1 and S2 fit within the base array at matrix locations a11 and a22 , but S3 breaches the array. However, you will notice that S3 is also the same as matrix element a13, because the 2D array is periodic. With S3 in matrix element position a13, we continue diagonally and S4 breaches the base array. However, S4 is also the same as base matrix element a21. Consequently, every time the base array is breached by the next diagonal sequence number, it is folded back into the base array. S5 is folded into matrix element a12 and S6 diagonally remains within the base array. The N=7 2D PRD is shown in Table 2.

$$S_n = r^n \bmod N; \;\; n = 1, 2, 3..N-1 \qquad \text{Eq. 1}$$

Table 1. Chinese remainder process

S1	S5	S3	S1	S5	S3
S4	S2	S6	S4	S2	S6
S1	S5	S3	S1	S5	S3
S4	S2	S6	S4	S2	S6
S1	S5	S3	S1	S5	S3
S4	S2	S6	S4	S2	S6

Table 2. N=7 2D PRD

3	5	6
4	2	1

To maximize the bandwidth of a diffusor, we need to make the size of the basic repeat unit as large as possible. In this case, we want to use a large prime and fold it into a larger, nearly square base array. You can practice folding an N=31 PR sequence into a 5x6 array and arrive at the pattern in Table 3.

Table 3. 2D PRD based on N=31

3	18	15	28	13	16
17	9	23	14	22	8
24	20	27	7	11	4
12	10	29	19	21	2
6	5	30	25	26	1

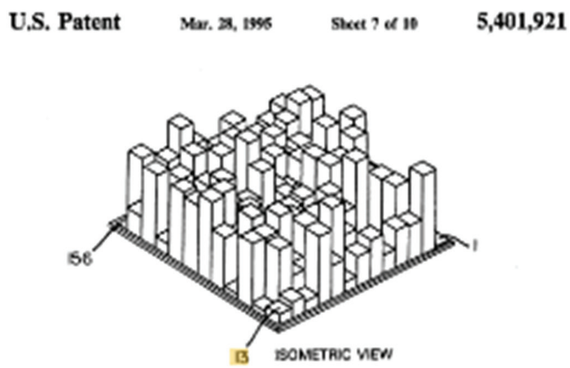

Figure 1. Skyline diffusor N=157

In 1995, RPG Diffusor Systems patented, Figure 1, the first PRD based on N=157 with 12x13 coprimes with nearly square block elements and introduced this new technology to the architectural acoustics community.

In the next post, we will explore some limitations of number theoretic reflection phase grating diffusers and begin to describe how RPG has resolved these issues with innovative research and development.

Peter D'Antonio

Dr. Peter D'Antonio
Director of Research
Acoustical Research Center