DiffuseReflections 220421

# Experimental Measurement Lab: Broad Bandwidth Impedance Tube

In Diffuse Reflections 220414, we described the ARC's low-frequency tube, which extends the measurement down to 20 Hz. To provide broad bandwidth normal incidence measurements, we developed a proprietary 160 mm x 160 mm (6.3" x 6.3") square tube that allows measurement from 63-4,000 Hz. This is an especially useful complement to the rev room because prototypes can be evaluated, using a much smaller sample compared to the rev room, and the impedance tube also determines the complex surface impedance $z_1$, which provides more design information than the simple absorption coefficient. The design utilizes a single sample size and three impulse measurements with microphones at three specific positions from the sample, using a rigid or anechoic termination. This is a significant benefit compared to commercial tubes, which require several tube diameters to cover this range. A photo of this tube can be seen in Figure 1.

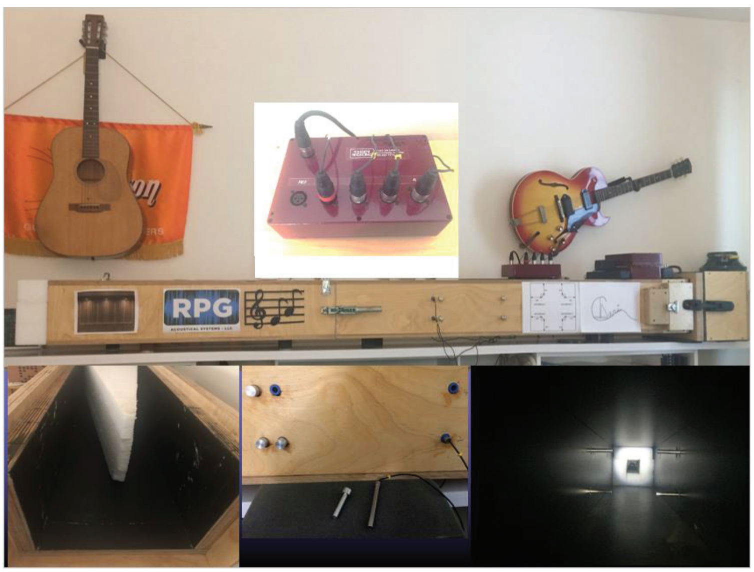



*Figure 1. **Top:** The impedance tube is shown with an anechoic termination. **Bottom left:** Anechoic termination wedge. **Bottom center:** Three microphone positions with one microphone and plug. **Bottom right:** View down the tube toward the source showing four microphones extending into the tube at 1/4 of the width and height.*

The highest frequency, $f_u$, that can be measured in a tube is determined by:

$$f_u = \frac{c}{2d}$$

where d is the tube diameter or maximum width and c is the speed of sound. This is a statement that there should not be any cross modes in the tube; the first mode appears when half of a wavelength fits across the tube. This high-frequency limitation means that to cover a wide-frequency range, several different impedance tubes of different diameter or width are required. However, it is possible to measure to higher frequencies with one tube if multiple microphones are used across the width of the tube. This technique is exploited in the 160 mm square tube, by using four microphones placed as shown in Figure 2. To cover the full bandwidth, the three impulse responses, shown in Figure 3, are summed.

In this case, the first and third cross-modes in each direction cancel and the second mode is at a null, leaving the fourth order mode to dominate. This quadruples the high-frequency limit. Thus, the 160 mm square tube can measure up to the 4,000 Hz third octave band.

If plane waves are assumed to propagate in the tube, then the steady state pressure in the tube is given by:

$$p = A(e^{jkz} + Re^{-jkz})$$

where R is the reflection coefficient; k is the wavenumber; the sample is assumed to be at z=0, A is a complex constant. The first term represents the incident wave, and the second the reflected wave.

The steady state pressure equation has two unknowns, the magnitude and phase of the reflection coefficient. By measuring the pressure at two points in the tube, it is possible to set up and solve simultaneous equations for the reflection coefficient and from there get the impedance and absorption coefficient. This is the principle of the transfer function method.

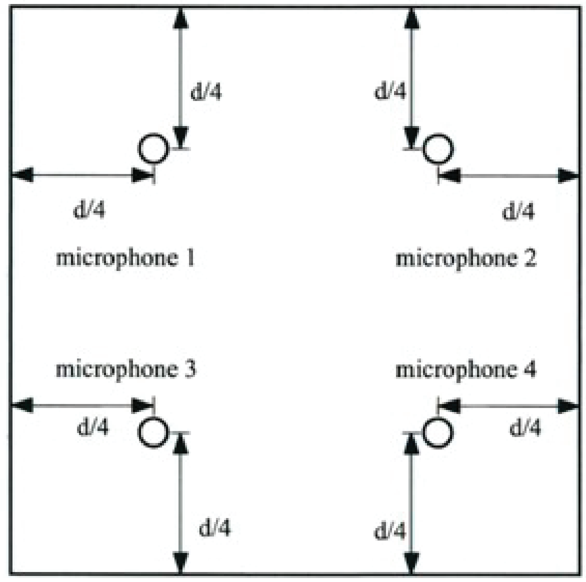



*Figure 2. Multi-mic setup to quadruple upper-frequency limit used in 160 mm impedance tube, where d is the width and height of the square tube. This results in a useful range of 63-4000 Hz.*

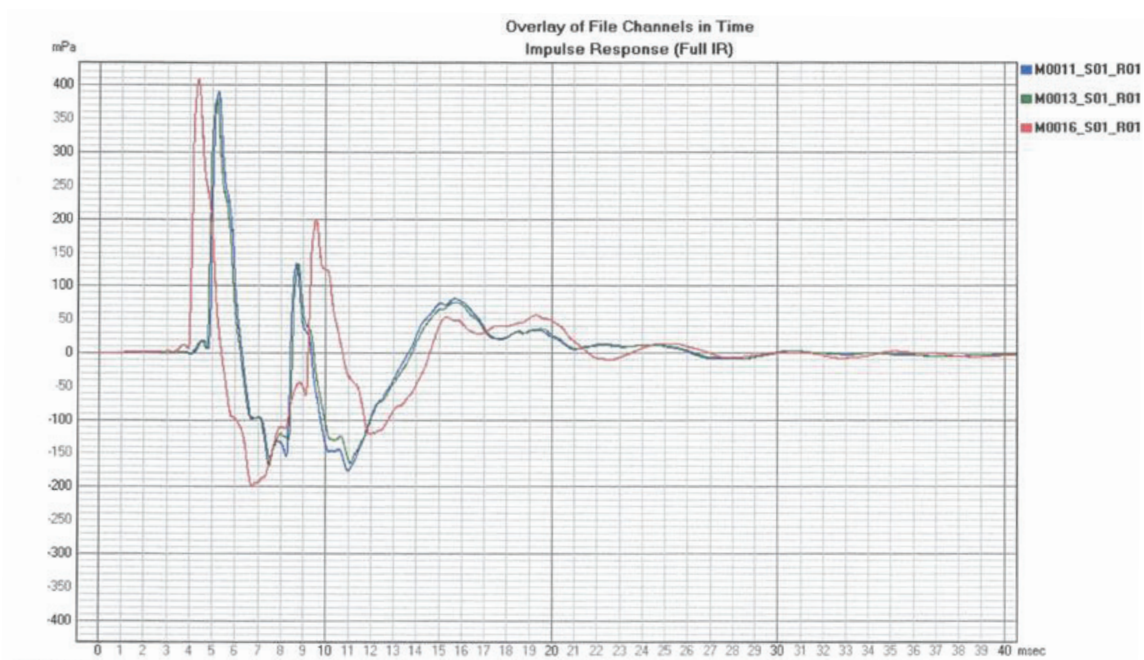



*Figure 3. Impulse responses at three specific positions for a 100 mm (4”) melamine sample with a rigid termination.*

The primary advantage of using this approach is that it obtains the complex impedance and absorption coefficient of the surface with only three quick measurements. The steady state pressure between two microphone positions in the tube is measured. Remembering that the transfer function is simply the ratio of pressures, $H_{12} = p(z_2)/p(z_1)$, and applying the steady state pressure equation, the transfer function between microphone positions 1 and 2 is given by:

$$H_{12} = \frac{e^{jkz_2} + \mathrm{R}e^{-jkz_2}}{e^{jkz_1} + \mathrm{R}e^{-jkz_1}}$$

where $z_1$ and $z_2$ are the positions of the four microphones shown in Figure 2. Rearrangement then directly leads to the complex pressure reflection coefficient:

$$R = \frac{H_{12}e^{jkz_1} - e^{jkz_2}}{e^{-jkz_2} - H_{12}e^{-jkz}}$$

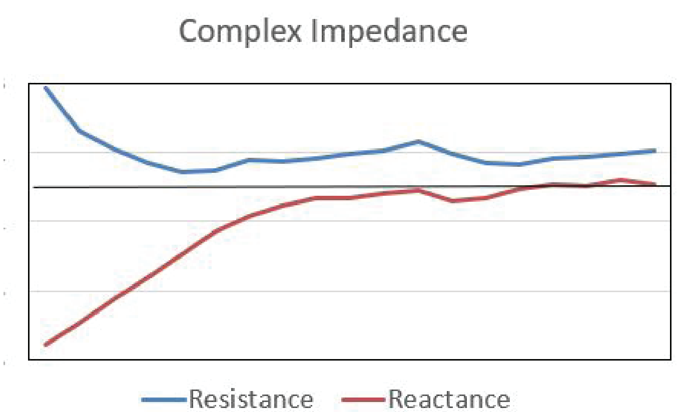
The need for continuity of particle velocity normal to the surface enables the derivation of an expression for the specific acoustic impedance of the surface, which is normalized by the characteristic impedance of air **ρc**. The relationships between pressure reflection coefficient and impedance for normal incidence is given by:

$$\frac{z_1}{\rho c} = \frac{1+R}{1-R}$$

The absorption coefficient, **α**, is a ratio of the absorbed and incident energy, enabling the following expression to be derived:

$$\alpha = 1 - |R|^2$$

where |R| is the magnitude of the pressure reflection coefficient.



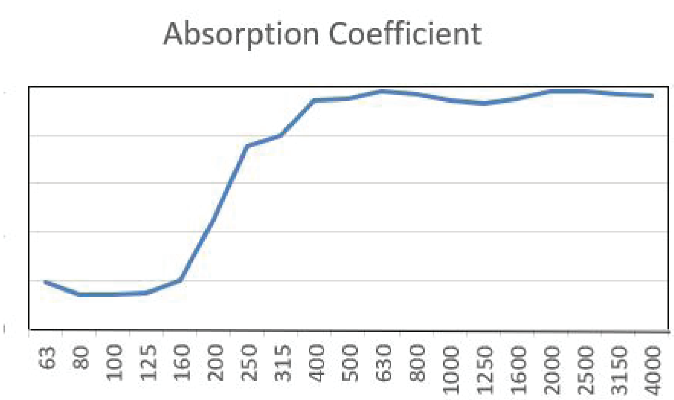




*Figure 4.* ***Top:*** *Complex specific impedance.* ***Bottom:*** *Normal incidence absorption coefficient of a 100 mm melamine sample.*

In Figure 4, we show the specific complex impedance and absorption coefficient for a 100 mm melamine sample. An analysis of the Resistance and Reactance of the complex specific surface impedance is instructive. The graph illustrates that the real part of the normalized impedance (Resistance) is essentially 1 down to roughly 400 Hz. This results in an absorption coefficient of essentially 1. Higher or lower Resistance values provide guidance as to the resistance of the sample offers to the incident wave. When the imaginary part of the normalized impedance (Reactance) crosses zero, a resonance occurs. If the Resistance is 1 at this frequency, then the resonant device will offer 100% absorption.

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