Virtual Education Lab: The QRD design procedure

In this post, we begin with a description of the reflection phase grating diffusor, which is based n number theory sequences that have a flat power spectrum. It is generally understood in signal processing that the Fourier transform of a delta function in the time domain results in a constant flat response in the frequency domain. Similarly, if the Fourier transform is applied to a delta function autocorrelation of the surface reflection coefficients, the result is a constant uniform distribution in the spatial domain. Since the 1-dimensional (1D) quadratic residue sequence (QR) is one of several sequences that possesses a delta function autocorrelation, it was selected to be used to design a quadratic residue diffusor (QRD), Figure 1, and was the first type of commercial sound diffusor that was manufactured and successfully tested.

The sequence generating function is as follows:

$$S_n = n^2 \bmod N$$

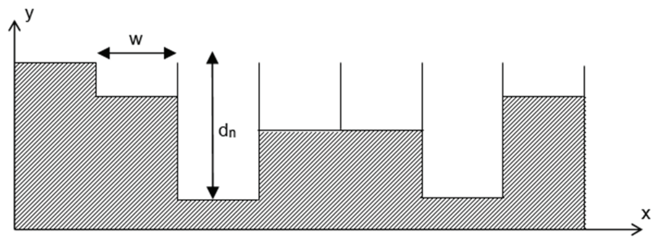

Figure 1. Schematic N=7 QRD diffusor

Where S_n is the sequence number, n is an integer, N is a prime number generator and the number of wells per period and mod (modulo) indicates the lease nonegative remainder. In Table 1, we generate the QR sequence with a prime N=7 and the well depths d_n in inches, for design frequencies f_0=1000 Hz and 500 Hz. It is important to realize that the QR sequence is periodic (blue), and repeats every 7 elements. The calculated well depths d_n are a function of the sequence values S_n, the speed of sound, c, the prime number, N, and the design frequency, f_0.

$$d_n = \frac{S_n c}{2 N f_0}$$

For the design theory to be correct, plane wave propagation in the wells must dominate. Consequently, the upper frequency determinds the width of a well, as shown in Figure 1, where f_{max} is the maximum frequency before cross modes in the wells appear. This requirement for plane wave propagation also explains the need for narrow dividers between the wells. For example, an f_{max} of 5,000 Hz requires a well width, w, of 1.36”.

$$w = \frac{c}{2 f_{max}}$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
n^2	0	1	4	9	16	25	36	49	64	81	100	121	144	169
mod 7	0	1	4	2	2	4	1	**0**	**1**	**4**	**2**	**2**	**4**	**1**
f_0=1000 Hz	0.00	0.97	3.87	1.94	1.94	3.87	0.97	0.00	0.97	3.87	1.94	1.94	3.87	0.97
f_0=500 Hz	0.00	1.94	7.75	3.87	3.87	7.75	1.94	0.00	1.94	7.75	3.87	3.87	7.75	1.94

Figure 1. QR sequence generation and well depth calculation (inches).

In summary, we have explained how to generate a QR sequence and used it to determine well depths for two design frequencies of 1,000 Hz and 500 Hz. We also determined the well width for a maximum frequency of 5,000 Hz. These limits are intended as design guides. Experimental measurements have demonstrated that uniform diffusion extends below the design frequency. In the next post, we continue our description of additional characteristics of reflection phase gratings.

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